Letters to the Editor

Indian J. Phys. **49**, 401–402 (1975)

On the solution of equations of connection in the new unified field theory

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(*Received* 5 *December* 1974)

In the previous note (Ghosh 1973) the equations of connection were obtained as

$$\partial_\sigma S_{\mu\nu} = S_{\lambda\nu}P^\lambda{}_{\mu\sigma} - F_{\lambda\nu}V^\lambda{}_{\mu\sigma} + S_{\lambda\mu}P^\lambda{}_{\nu\sigma} - F_{\lambda\mu}V^\lambda{}_{\nu\sigma} \qquad \ldots \quad (1)$$

$$\partial_\sigma F_{\mu\nu} = -S_{\lambda\nu}V^\lambda{}_{\mu\sigma} + F_{\lambda\nu}P^\lambda{}_{\mu\sigma} + S_{\lambda\mu}V^\lambda{}_{\lambda\sigma} - F_{\lambda\mu}P^\lambda{}_{\nu\sigma} \qquad \ldots \quad (2)$$

where

$$S_{\mu\nu} = \tfrac{1}{2}(g_{\mu\nu}+g_{\nu\mu}), \qquad F_{\mu\nu} = \tfrac{1}{2}(g_{\mu\nu}-g_{\nu\mu}),$$

$$P^\lambda{}_{\mu\nu} = \tfrac{1}{2}(\tilde{\Gamma}^\lambda{}_{\mu\nu}+\Gamma^\lambda{}_{\mu\nu}), \qquad V^\lambda{}_{\mu\nu} = \tfrac{1}{2}(\tilde{\Gamma}^\lambda{}_{\mu\nu}-\Gamma^\lambda{}_{\mu\nu}).$$

The 80 connection coefficients $\Gamma^\lambda{}_{\mu\nu}$, $\tilde{\Gamma}^\lambda{}_{\mu\nu}$ symmetric in μ, ν are restricted by the conditions

$$\Gamma^\lambda{}_{\mu\mu} = \tilde{\Gamma}^\lambda{}_{\mu\mu} \ (\mu \text{ not summed}) \qquad \ldots \quad (3)$$

$$\tilde{\tilde{\Gamma}}^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\mu\nu}.$$

From eq. (1) we obtain

$$S^\rho{}_{\mu\nu} = P^\rho{}_{\mu\nu} - S^{\rho\sigma}F_{\lambda\sigma}V^\lambda{}_{\mu\nu}. \qquad \ldots \quad (4)$$

By substituting eq. (4) in eq. (2) we have

$$\partial_\sigma F_{\mu\nu} - F_{\rho\nu}S^\rho{}_{\mu\sigma} + F_{\rho\mu}S^\rho{}_{\nu\sigma} = (S_{\lambda\mu} - F_{\rho\mu}S^{\rho\alpha}F_{\lambda\alpha})V^\lambda{}_{\nu\sigma} - (S_{\lambda\nu} - F_{\rho\nu}S^{\rho\alpha}F_{\lambda\alpha})V^\lambda{}_{\mu\sigma} \qquad \ldots \quad (5)$$

Denoting

$$S_{\lambda\mu} - F_{\rho\mu}S^{\rho\sigma}F_{\lambda\alpha} \text{ by } U_{\lambda\mu}$$

and noting $U_{\lambda\mu} = U_{\mu\lambda}$, eq. (5) is expressed as

$$F_{\mu\nu\sigma} = U_{\lambda\mu}V^\lambda{}_{\nu\sigma} - U_{\lambda\nu}V^\lambda{}_{\mu\sigma}, \qquad \ldots \quad (6)$$

where

$$F_{\mu\nu\sigma} + F_{\nu\sigma\mu} + F_{\sigma\mu\nu} = 0. \qquad \ldots \quad (7)$$

Referring to the field equations proposed in the previous note (Ghosh 1973) we have to incorporate the set of auxiliary equations further

$$V^\sigma{}_{\mu\sigma} = 0, \qquad P^\sigma{}_{\mu\sigma} = \partial_\mu \log(-g)^{\frac{1}{2}}. \qquad \ldots \quad (8)$$

We have now the system of eqs. (6) and (8) to determine V's while the P's are given by eq. (4).

Let us apply the above formulae to the case of a special non-symmetric tensor field $g_{\mu\nu}$ with symmetric components S_{11}, S_{22}, S_{33}, S_{44} and the anti-symmetric components F_{14}, F_{23}, F_{32}, F_{41}, where all the components are functions of four variables v_1, v_2, v_3, v_4. In the non-symmetric field theory of Einstein this case has been tackled in a more complicated way to obtain the solution of non-symmetric Γ's (Ghosh 1955).

We first note that the surviving components of $U_{\lambda\mu}$ in eq. (6) are

$$\begin{aligned} U_{11} &= S_{11}\,(1-C_4^{\,1}C_1^{\,4}), & U_{22} &= S_{22}\,(1-C_3^{\,2}C_2^{\,3}), \\ U_{33} &= S_{33}\,(1-C_3^{\,2}C_2^{\,3}), & U_{44} &= S_{44}\,(1-C_4^{\,1}C_1^{\,4}), \end{aligned} \tag{9}$$

where $C^\lambda{}_\mu$ denotes $F_{\lambda\mu}/S_{\lambda\lambda}$.

From eq. (3) one gets $V_{\mu\mu}{}^\lambda = 0$, (μ, not summed). Using the above we now frame from eq. (6) the following set of equations

$$\begin{aligned} F_{1\nu 1} &= U_{11}V_{\nu 1}{}^1, & F_{3\nu 3} &= U_{33}V_{\nu 3}{}^3, \\ F_{2\nu 2} &= U_{22}V_{\nu 2}{}^2, & F_{4\nu 4} &= U_{44}V_{\nu 4}{}^4. \end{aligned} \tag{10}$$

Rest of the equations appear as follows

$$\begin{aligned} F_{123} &= U_{11}V_{23}{}^1 - U_{22}V_{13}{}^2, & F_{134} &= U_{11}V_{34}{}^1 - U_{33}V_{14}{}^3 \\ F_{231} &= U_{22}V_{31}{}^2 - U_{33}V_{21}{}^3, & F_{341} &= U_{33}V_{41}{}^3 - U_{44}V_{31}{}^4, \\ F_{124} &= U_{11}V_{24}{}^1 - U_{22}V_{14}{}^2, & F_{234} &= U_{22}V_{34}{}^2 - U_{33}V_{24}{}^3 \\ F_{241} &= U_{22}V_{41}{}^2 - U_{44}V_{21}{}^4, & F_{342} &= U_{33}V_{42}{}^3 - U_{44}V_{32}{}^4. \end{aligned} \tag{11}$$

To solve for P's we have now the system of eq. (4)

$$P_{\mu\mu}{}^\lambda = S_{\mu\mu}{}^\lambda = \Gamma_{\mu\mu}{}^\lambda.$$

$$\begin{aligned} P_{\mu\nu}{}^1 &= S_{\mu\nu}{}^1 - C_4^{\,1}V_{\mu\nu}{}^4, & P_{\mu\nu}{}^3 &= S_{\mu\nu}{}^3 - C_2^{\,3}V_{\mu\nu}{}^2, \\ P_{\mu\nu}{}^2 &= S_{\mu\nu}{}^2 - C_3^{\,2}V_{\mu\nu}{}^3, & P_{\mu\nu}{}^4 &= S_{\mu\nu}{}^4 - C_1^{\,4}V_{\mu\nu}{}^1. \end{aligned} \tag{12}$$

It appears that in general four of the V's remain undetermined, so also four of the P's corresponding to them.

References

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Ghosh N. N. 1973 *Ind. J. Phys.* **47**, 187.